

# Clustering Evolutionary Stock Market Model

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(Dated: February 2, 2008)

As a typical representation of complex networks studied relatively thoroughly, financial market presents some special details, such as its nonconservation and opinions spreading. In this model, agents congregate to form some clusters, which may grow or collapse with the evolution of the system. To mimic an open market, we allow some ones participate in or exit the market suggesting that the number of the agents would fluctuate. Simulation results show that the large events are frequent in the fluctuations of the stock price generated by the artificial stock market when compared with a normal process and the price return distribution is a *lévy* distribution in the central part followed by an approximately exponential truncation.

PACS numbers: 87.23.Ge, 02.50.Le, 05.45.Tp, 05.65.+b

## I. INTRODUCTION

As the research of the complex systems is getting deeper and deeper, to find the universal rules and principles of these systems and to answer the origination of “complexity” become more and more attractive [1, 2, 3, 4, 5, 6]. Thus, the vision of physicists is no longer restricted in the traditional areas but concentrates on the more comprehensive domains, leading to the birth of many burgeoning disciplines through the interaction and amalgamation of physics and other fields such as biology, finance, sociology, and so on. Over the last decade, there has been significant interest in applying physical methods in social and economical science [7]. In particular, the study of financial market prices has been found to exhibit some universal properties similar to those observed in physical systems with a large number of interacting units, and some models [8, 9, 10, 11, 12] (as we know, the first stock market simulation was performed by economist Stigler in 1964 [13]) have been introduced to the financial and more recently to the physical communities which attempt to capture the complex behaviors of stock market prices and market agents [14]. These models, including behavior-mind model [15, 16], dynamic-games model [17], multi-agent model [18, 19, 20, 21, 22, 23] and so on, are based on the statistical properties [24] of price fluctuations which should be recovered by more suitable microscopic models [25]: (1) sharp peak and fat-tail distribution for the price changes (the return histogram) [26]; (2) the distribution of returns decays with power law in the tails [27], with exponent near 3; (3) price fluctuations are not invariant against time reversal, i.e. they show a forward-backward asymmetry [28].

Among the more sophisticated approaches are dynamic multi-agent models based on the interactions of two distinct agent populations (“noisy” and “fundamental” traders), which could simulate the price forming processes and reproduce some of the stylized observations of real markets, but fail to account for the origin of the universal characteristics. An alternative approach, the herd behavior [29, 30] explored in this paper, may be capable to induce the power-law asymptotic behavior in the tail of price return distribution with an exponent well fitting the truncated Lévy regime [26] as found in real data [11, 27]. This approach has been formalized by Cont and Bouchaud [31] as a static percolation model. Subsequently, this percolation model has been bettered by introducing a feedback mechanism between the price return  $Z$  and trader activity  $a$ :  $a \rightarrow a + \alpha Z$ , where  $\alpha$  is the factor denoting the sensitivity to price fluctuations. Then the volatility clustering can be reproduced, and all of the statistical properties of fluctuations for prices mentioned above could be observed.

In the Cont-Bouchaud model, random percolation clusters are used as groups of traders. In the simple version, at each iteration each cluster buys with probability  $a$ , sells with probability  $a$  or sleeps with probability  $1 - 2a$ . When

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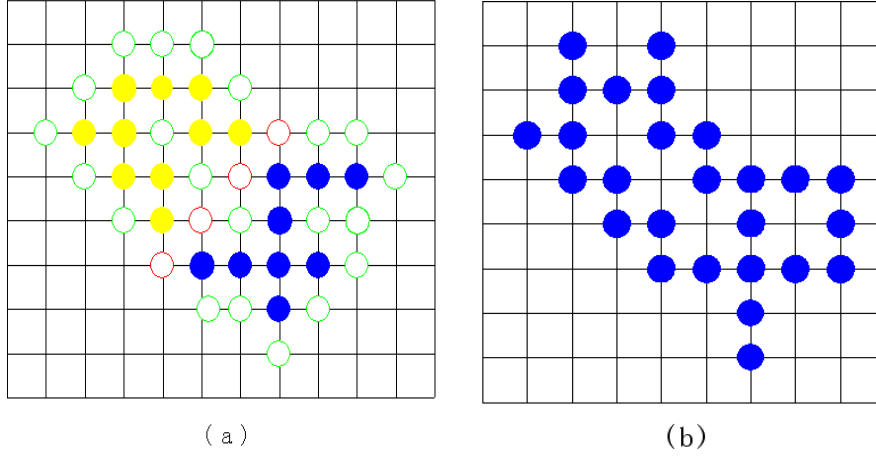


FIG. 1: (a) To illustrate, this is a small-scale matrix comparing to our model which can help us to explain what is a cluster called in this paper. A cluster, here we call it  $M$ , is defined as an aggregation of nodes that share the same information and hold the same opinion. Moreover, in the topology, every two nodes in  $M$  can reach each other. Obviously, there are two different clusters which are represented by different colors, yellow and blue in figure 1(a). In the process of growth, each cluster has distinct probability to absorb new members who could occupy the frontier and empty nodes with different probabilities determined by the cluster which they would participate in. And the sites which the new agents could occupy are denoted by the hollow nodes. But the colors of the hollow nodes are distinguished which implies that there must be some difference between the positions marked by different colors. In fact, the positions denoted by red hollow nodes mean that if a new member comes into the market and takes up one of these positions, he would be puzzled for there are two different attitudes. In other words, he comes across two different information resources which would lead to completely different consequences. And he operates just like a information bridge which would stimulate the communication of clusters holding different attitudes and even lead to a serious confliction that could promote some certain clusters to merge others and expand to a giant one such as the cluster shown in figure 1(b). The opposite process called "collapse of the cluster" in this model, which means that the nodes belonging to a same cluster are removed from the lattice, reflects another phenomenon of the real market that there are always some traders quitting the market due to different reasons which may lead to a serious influence on the price.

the activity  $a$  is small, there are only few clusters trading at a time most of the times. Therefore, the distribution  $P_b(Z)$  of relative price fluctuations or "returns"  $Z$  scales as the well-known [32, 33, 34] cluster size distribution of percolation theory. But when we increase the value of  $a$ , more and more clusters would make contribution to the relative price fluctuations and the central limit theorem suggests that the distribution  $P_b(Z)$  convert from power-law tails to a more Gaussian shape for large systems. Price changes in the logarithm (return) are proportional to the difference between the supply and demand. On average, price rises or falls with equal probability and without correlations between consecutive steps. An assumption made in this model which should not be ignored is that the probability  $a$  (activity) is set to be the same for all groups and remains constant through out the whole process, which may be a good strategy for simplifying a physical model but may be not a good regulation for establishing a model which we expect to reflect the various phenomena found in the real stock market as genuine as possible so it could be more helpful for us to capture the complex properties of the real world. Although in the successional studies of the percolation model different mechanisms are used to establish a self-organized model where the investor groups with various trading activities and sizes are formed automatically, few of these models considered the fluctuations of the traders in the market, fact that there are always agents who take part in or exit from the market due to various reasons which might have a serious influence on the price. Here we introduce a self-organized model where the activities and sizes of different investor groups are driven by the confliction and harmonization of the strategies adopted by different groups. The simulation results which well agree with the observations of real markets are also shown in the third section.

## II. THE NEW MODEL

Considering an open market which absorbs and removes traders with probabilities respectively in the process of trading, we draw the growth and collapse of clusters into our model based on Cont-Bouchaud model's random percolation. After every  $N$  iterations of trading, each cluster which is defined in figure 1 grows around itself (figure 1(a)) with probability  $P_d$ , collapses and annihilates with probability  $P_n$ , or sleeps with probability  $1 - P_d - P_n$ . Once

a cluster collapses, some new clusters will come into the market at the positions where the collapse occurred (figure 1(b)), with a fixed probability  $P_h$ . We build our model as follows based on the thoughts above:

(1) Initiation: a  $L \times L$  lattice is occupied randomly with probability  $P_{in}$ , and each cluster is randomly given a state: buying, selling, or sleeping, which are represented by 1, -1 and 0 respectively.

(2) Trading: at time  $t$ , each agent in the market sells or buys a unit of stock. Then we calculate the difference between the supply and demand

$$r = \sum_{i=1}^m s_i \quad (1)$$

where  $m$  is the total number of agents who are presented in the market at  $t$  and  $s_i$  represents the state of the  $i$ -th agent. The evolution of price follows the rule:

$$P(t+1) = P(t)e^{r/\lambda} \quad (2)$$

where  $\lambda = N_p + N_n$ . Here  $N_p$  and  $N_n$  denote the number of buyers and sellers, respectively. Figure 2 shows the price time series, which is rather similar to that of real stock market. And the active probability  $2a$  with which the agents choose buying or selling rather than sleeping evolves following the Equ.(3):

$$a(t) = a(t-1) + lr \quad (3)$$

where  $l$  represents the sensitivity of activity  $a(t)$  to the difference between the demand and supply. And then, each cluster buys, sells, or sleeps with probabilities  $2a(t)p_b$ ,  $2a(t)p_s$ ,  $1 - 2a(t)$  respectively, where

$$p_b = \begin{cases} \mu + \nu_1 r, & r < 0 \\ \mu + \nu_2 r, & r > 0 \end{cases} \quad (4)$$

and

$$p_s = 1 - p_b \quad (5)$$

The first term on the right hand of both of Equ.(4) denotes the probability with which the active one would buy rather than sell without considering the feedback of the price fluctuations. The difference between the case  $r < 0$  and  $r > 0$  is the coefficient of the last term on the right hand and takes into account that agents are risk adverse and thus more impressed by a downturn than by an upturn of the market so that the parameters  $\nu_1$  and  $\nu_2$  denote the sensitivity of the agent's mentality to the price fluctuations. (The price would fall if  $r < 0$  and would rise if  $r > 0$  according to Equ.(2) [35, 36, 37, 38, 39, 40]). So the value we adopt of  $\nu_1$  is smaller than that of  $\nu_2$  [41].

(3) Growth: after each  $N$  iterations, there are three types of evolution with different probabilities respectively depicted in the following segments: growing, collapsing and sleeping.

The first situation is that new traders come into the market occupying the empty sites around the old clusters, for example, cluster **G**, just as the sites marked by hollow nodes proposed in figure 1(a) with the probability

$$P_d^g(t+1) = P_d^g(t) + k(N_T - c^g(t)) \quad (6)$$

where  $k$  is a kinetic coefficient and  $N_T$  is a threshold parameter [42], and

$$c^g(t) = \sum_{j=1}^{m_g} (|s_j^g|) \quad (7)$$

in which  $m_g$  represents the scale of the cluster **G**, in other words, the number of nodes which belong to **G**.  $s_j^g$  denotes the state of the  $j$ -th node belonging to **G**. The probability  $P_d^g(t+1)$  is obviously limited to the range  $[0, 1]$  so that we have to impose  $P_d^g(t+1) = 0$  and  $= 1$  if the recurrence relationship Equ.(6) gives values for  $P_d^g(t+1) < 0$  or  $> 1$ . If a few of different clusters whose states are different encounter, one (noted by **V**) will defeat others (the total number is  $n$ , including **V**) with the probability

$$P_v(t) = |c^v(t)| / \sum_{i=1}^n |c^i(t)| \quad (8)$$

And the evolution due to Equ.(8) would lead to the consequence that the defeated clusters would accept the opinion and adopt the same strategy of the winner. In other words, they are annexed by the winner. By contrary, when the states of the encountered clusters are all the same, they would combine and operate as a whole.

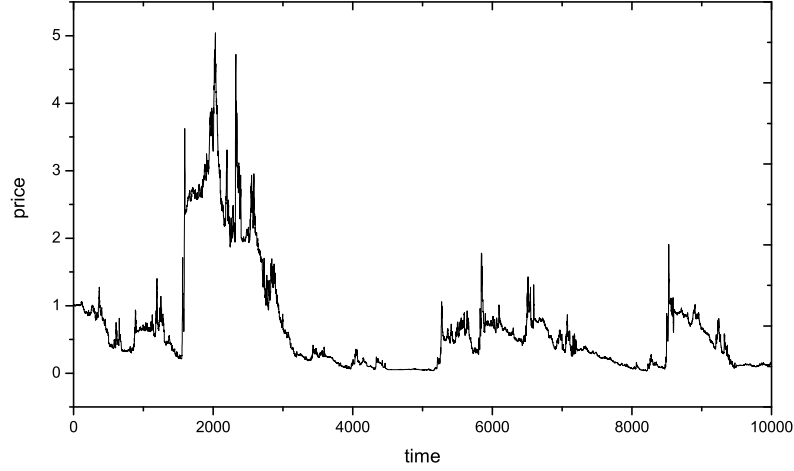


FIG. 2: Time series of the typical evolution of the stock price, where  $P_h=0.01$ . One can see that the trend and fluctuations of the stock price generated by our model are rather similar to that of real stock market.

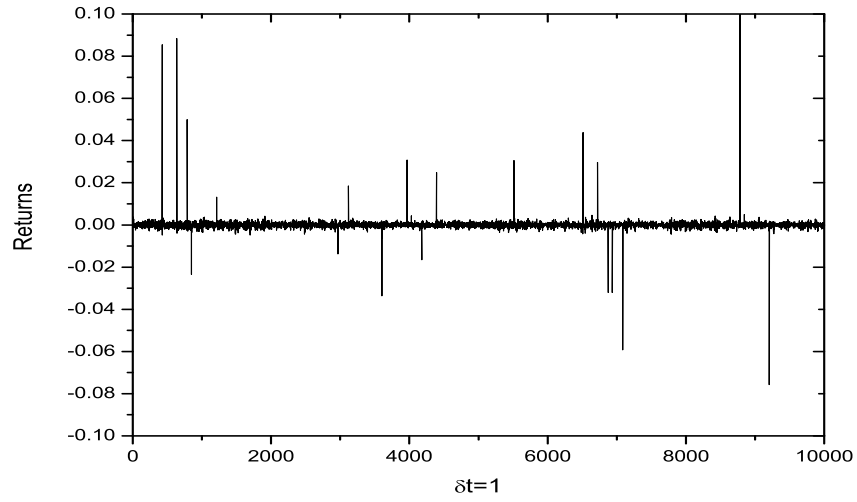


FIG. 3: The returns of simulated price fluctuations for  $\delta t=1$ . It can be seen that large events are frequent in the fluctuations of stock market when compared with a normal process.

The second, each old cluster such as  $\mathbf{Q}$  collapses with the probability

$$P_n^q(t) = c^q(t)/L^2 \quad (9)$$

which indicates that the probability with which a cluster collapses would increase with its growth. When a cluster takes up all of the sites of the lattice, it would surely collapse. Once the old cluster collapses, the members of the new clusters whose states are not necessarily the same as the old one are produced with fixed probability  $P_h$  and take up the sites where the old cluster has existed.

The final circumstance, clusters sleep with the probability  $1 - P_d(t) - P_n(t)$ .

(4) Repeat step (2) and (3) for enough times.

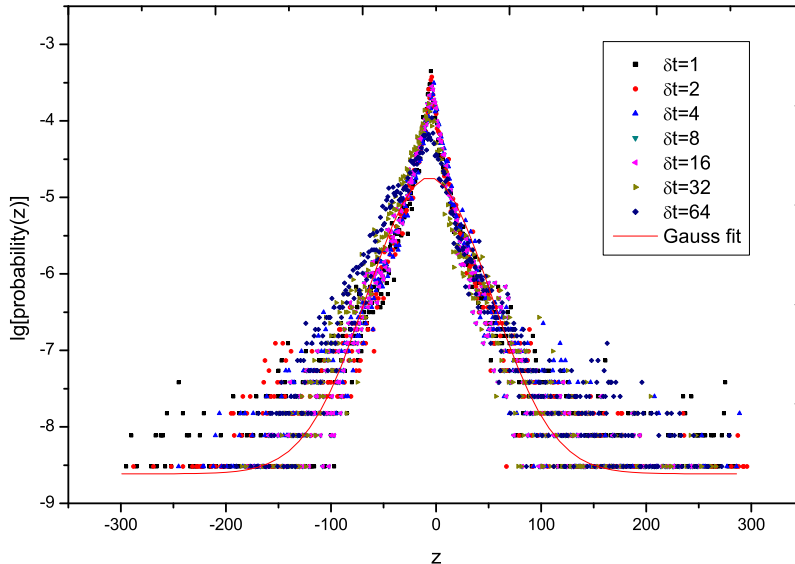


FIG. 4: The probability distributions of price returns with  $\delta t=1, 2, 4, 8, 16, 32, 64$  respectively. In this figure, the central part of the distribution of returns appears to be well fitted by a lévy distribution.

### III. SIMULATION RESULTS

The typical parameter space we adopt in our simulation is as follows:  $l = 0.0001$ ,  $k = 0.0001$ ,  $N_T = 50$ ,  $P_h = 0.01$ ,  $a(0) = 0.35$ ,  $P_d(0) = 1.0$ ,  $N = 100$ ,  $\mu = 0.59$ ,  $\nu_1 = 0.00005$  and  $\nu_2 = 0.0001$ . About 100 traders (in other words,  $P_{in}=0.01$ ) are distributed on a square lattice randomly and the initial stock price is 1.0. The simulation results are very sensitive to some of the parameters such as  $l$ ,  $k$ ,  $\mu$ ,  $\nu_1$  and  $\nu_2$ . When the values of them are little larger, the price fluctuations would be very exquisite and when they are little smaller, the price trend would be very meek. But the simulation results are not very sensitive to other parameters for other values of them could lead to the results which are in good agreement with the real data, too. But there are some amazing results we should point out: (1)  $\mu$  is not 0.5 but 0.59, little larger than 0.5, which is very close to the threshold value: 0.593 in the percolation theory which may be a support to the point that the real markets should operate close to the critical point where profitable trade opportunities are barely detectable [43]. The process by which the market self-organizes close to the critical point is more likely to be of evolutionary nature and hence to take place on longer time scales [44]. And this result suggests that the choice whether to buy or to sell is not completely random as the traditional point stands which implies that  $\mu$  should be 0.5. (2) From the value space we could see that  $\nu_2 = 2\nu_1$ , which means that the affliction which is brought by losing one unit of wealth would be twice as much as the satisfaction which is brought by gaining the same amount of wealth according to Kahneman's Prospect Theory [40].

To compare the statistical properties of the price generated by our model and that of real stock markets further, we study the returns of price which is defined as Equ.(10):

$$Z(t) = \log P(t + \delta t) - \log P(t) \quad (10)$$

Mandelbrot has proposed that the distribution of returns is consistent with a Lévy stable distribution [45]. In 1995, Mantegna and Stanley analyzed a large set of data of the S&P500 index. It has been reported that the central part of the distribution of S&P500 returns appears to be well fitted by a Lévy distribution, but the asymptotic behavior of the distribution shows faster decay than that predicted by a Lévy distribution [26, 46]. The similar characteristic of the distribution of returns is also found in Hang Seng index [47]. Figure 4 shows the probability density of normalized returns, which display a clear Lévy distribution for  $\delta t=1, 2, 4, 8, 16, 32, 64$ .

Because larger  $\delta t$  implies less data points, it is difficult to determine the parameters characterizing the distributions only by investigating the spreads. Hence, we studied the peak values at the center of the distributions, i.e., the probability of zero return  $P_b(Z = 0)$  as the function of  $\delta t$ . With this choice, we can investigate the point of each

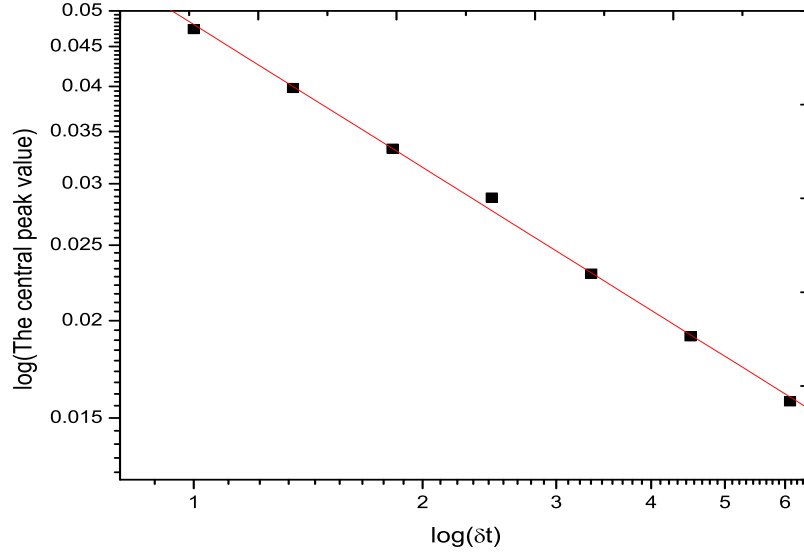


FIG. 5: The central peak value as a function of  $\delta t$ . The slope of the fitted line is  $-0.61 \pm 0.01$  which is very close to the real value 0.62 found in Hang Seng index [47].

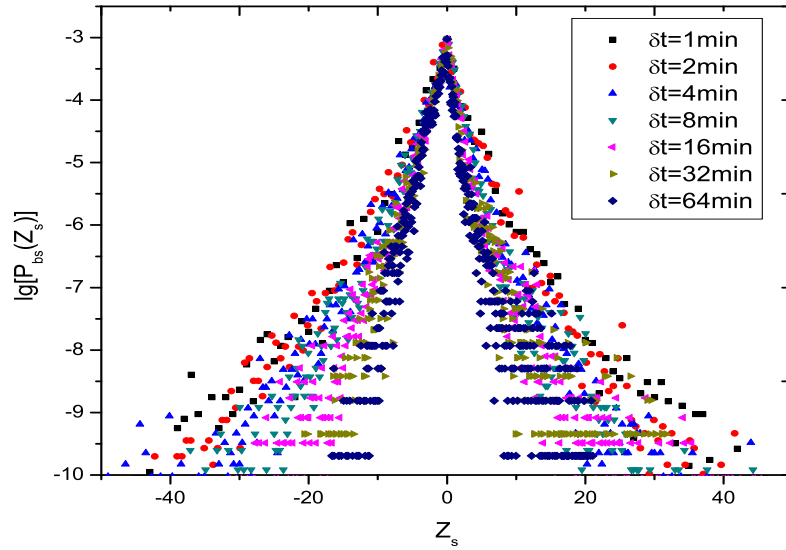


FIG. 6: Re-scaled plot of the probability distributions shown in figure 4. Data collapse is evident after using re-scaled variables with  $\alpha = 1.61$ . The abscissa is the re-scaled returns, the ordinate is the logarithm of re-scaled probability.

probability distribution which is least affected by noise. Figure 5 shows the central peak value versus  $\delta t$  in a log-log plot. It can be seen that all the data is well fitted by a straight line with the slope  $-0.61 \pm 0.01$  which is very close to the real value 0.62 found in Hang Seng index [47]. This observation agrees with theoretical model leading to a Lévy distribution.

If we assume that the central part of the distribution of returns can be described by a Lévy stable symmetrical

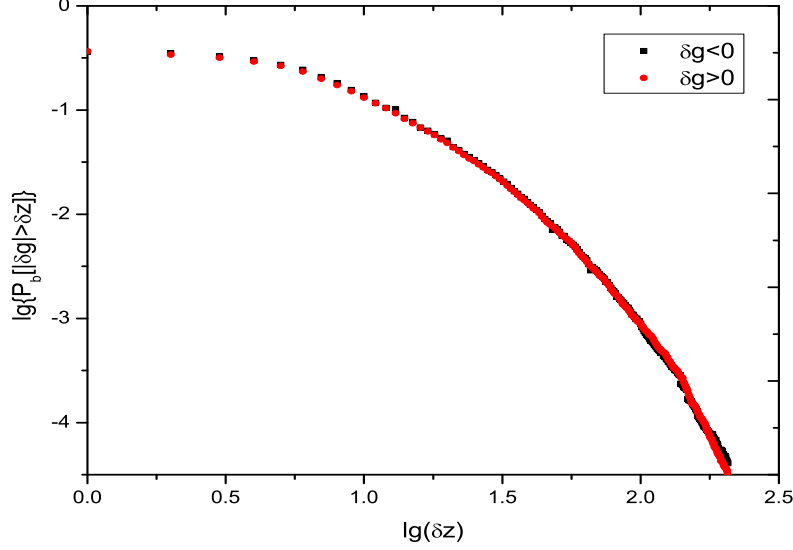


FIG. 7: The accumulate probability distributions  $P_b(g > Z)$  of 1-minute returns generated by our model. For data in the region  $10 \leq Z \leq 200$ , regression fits yield  $\alpha = 2.93$  (positive tail) and  $\alpha = 2.78$  (negative tail).

distribution with an index  $\alpha$  and parameter  $\gamma$ ,

$$P_b^\alpha(Z, \delta t) \equiv (1/\pi) \int_0^\infty e^{-\gamma \delta t |q|^\alpha} \cos(qZ) dq \quad (11)$$

where  $e^{-\gamma \delta t |q|^\alpha}$  is the characteristic function of a Lévy symmetrical stable process, the probability of zero return is given by

$$P_b(0) = P_b^\alpha(0, \delta t) = \Gamma(1/\alpha) / [\pi \alpha (\gamma \delta t)^{1/\alpha}] \quad (12)$$

where  $\Gamma$  is the Gamma function. Using the value  $-0.61 \pm 0.01$  for the slope of the fitted line to the data (figure 5), we obtain the index  $\alpha = 1.61 \pm 0.02$ .

To check whether the Lévy scaling can be extended to the entire probability distribution of returns generated by our model, we notice that under the transformation:

$$Z_s \equiv Z / [(\delta t)^{(1/\alpha)}] \quad (13)$$

and

$$P_{bs}(Z_s) \equiv (\delta t)^{1/\alpha} P_b^\alpha(Z, \delta t) = (\delta t)^{1/\alpha} P_b^\alpha[(\delta t)^{1/\alpha} Z_s, \delta t] \quad (14)$$

the distributions for different time scales  $\delta t$  will collapse onto one curve. Figure 6 shows the re-scaled distributions for the same data in figure 4 in the scaled variables, i.e.,  $P_{bs}(Z_s)$  versus  $Z_s$ . Data collapse is evident, except for some data points in the tails for large  $\delta t$ . The closer to the central point  $Z_s = 0$ , the stronger is the extent of data collapse. These observations imply that the Lévy distribution is a better description of the dynamics of the random process underlying the variation of returns in the central part of the probability distribution  $P_b(Z)$  over  $\delta t$  spanning at least two orders of magnitude.

In order to determine if an exponential truncated Lévy distribution can be used to describe the stochastic process and to investigate the kind of asymptotic behavior outside the Lévy stable region, we study the accumulate distribution  $P_b(g > Z)$  of the fluctuations of financial data.

For a stable symmetric Lévy distribution ( $0 < \alpha < 2$ ), the two tails show a power-law asymptotic behavior

$$P_b(Z) \sim Z^{-(1+\alpha)} \quad (15)$$

and hence the second moment diverges. This leads to an asymptotic power-law for the accumulate distribution for both the positive and negative tails [45] in the form

$$P_b(g > Z) \sim Z^{-\alpha} \quad (16)$$

Figure 7 shows the accumulate probability distribution of returns  $P_b(g > Z)$  for  $\delta t = 1$  min. for the data generated by our model. For data in the region  $10 \leq Z \leq 200$ , regression fits yield  $\alpha = 2.93$  (positive tail) and  $\alpha = 2.78$  (negative tail). These results appear to be outside the Lévy stable range of  $0 < \alpha < 2$  but they fit well the result produced from the real data which is near 3.

#### IV. CONCLUSION

Compared with the Cond-Bouchaud percolation model, our model presents a nonconservation market. With the evolution of the model's topology, there are new traders coming in and old ones leaving, which depicts the real stock markets more approximately. What is more, the process of the amalgamation and expansion, and the breakdown of the clusters in our model well consists with the phenomena (so called herd behaviors) in the real stock market that more and more people would take the same performance when they found more and more people around them take the same action. Some other simulations show that the information entropy, when we consider the clusters as various information resources and the process of merging and collapsing as the spreading and dying out of the information, has some relationships with the point which we define as the break-point of price. Moreover, there are also some amazing facts in the results of our simulations such as why the first term on the right hand of Equ.(4) approximate to the threshold value in the percolation theory. Since the main goal of this article is to establish and describe the model itself, we would not give detailed simulation results and analysis, which will be given elsewhere soon.

#### V. ACKNOWLEDGEMENT

This work has been partially supported by the National Natural Science Foundation of China under Grant No.70171053 and No.70271070, the Specialized Research Fund for the Doctoral Program of Higher Education (SRFDP No.20020358009), and the Foundation for graduate students of University of Science and Technology of China under Grant No. USTC-SS-0501.

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